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Black rings, supertubes, and a stringy resolution of black hole non-uniqueness

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Abstract: In order to address the issues raised by the recent discovery of non-uniqueness of black holes in five dimensions, we construct a solution of string theory at low energies describing a five-dimensional spinning black ring with three charges that can be interpreted as D1-brane, D5-brane, and momentum charges. The solution possesses closed timelike curves (CTCs) and other pathologies, whose origin we clarify. These pathologies can be avoided by setting any one of the charges, e.g., the momentum, to zero. We argue that the D1-D5-charged black ring, lifted to six dimensions, describes the thermal excitation of a supersymmetric D1-D5 supertube, which is in the same U-duality class as the D0-F1 supertube. We explain how the stringy microscopic description of the D1-D5 system distinguishes between a spherical black hole and a black ring with the same asymptotic charges, and therefore provides a (partial) resolution of the non-uniqueness of black holes in five dimensions.

Keywords: D-branes, Black Holes in String Theory, Black Holes.

Contents

1. Introduction

Rotating black holes typically exhibit much richer dynamics than their static counterparts, especially in more than four dimensions. Static black holes present relatively small qualitative differences as we increase the number of dimensions beyond four, but the inclusion of rotation brings in a larger diversity that depends on the dimensionality of spacetime [1]. A recent surprise has been the discovery of a rotating black ring solution in five-dimensional vacuum gravity [2]. The horizon of the black ring has topology $S^2 \times S^1$, and its tension and self-gravitational attraction are precisely balanced by the rotation of the ring. Perhaps more strikingly, when the mass and spin are within a certain range of values, it is possible to find a black hole and two black rings with the same spin and mass.[1] Hence, the simplicity of four-dimensional black holes implied by the celebrated uniqueness theorems does not extend to five-dimensional stationary solutions. It is quite possible that the non-uniqueness of higher-dimensional black holes is indeed wider than indicated by the existence of the black ring solution [3, 4].

[1] We follow [2] and often use the term "spherical black hole", or even simply "black hole", to abbreviate the phrase "topologically spherical black hole", as opposed to the black ring. Technically speaking, the black ring is also a black hole, but we hope to cause no confusion.

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String theory has provided a remarkably successful description of black holes, including an account of their entropy in microscopic terms [5], so it is natural to investigate in this context the implications of these novel features of black holes. In particular we can infer a striking consequence: to consistently account for these new solutions, there must exist string states that so far have gone unnoticed in the microscopic analysis of certain configurations with fixed conserved charges.

Our current understanding of neutral black holes within string theory is based on the "correspondence principle" [6]. According to it, a black hole is identified with a highly excited string state that is obtained by adiabatically decreasing the gravitational (string) coupling. When the black hole horizon shrinks to string-scale size, stringy corrections to gravity become too large to be neglected. At this point, the black hole can be matched to a string state with (parametrically) the same value for the entropy. So one may ask what are the string states that correspond to a black hole and to a black ring with the same spin and mass, and how they differ. Conversely, if we take a highly excited string and increase the coupling, at some point gravitational collapse will occur. For certain values of the initial mass and spin, the string will be confronted with the dilemma of what object it collapses into: a black hole or a black ring.

Unfortunately, the details of the string/black hole transition are still too poorly understood to enable us to see how, or even whether, string theory can distinguish between a neutral black hole and a black ring. The problem is further compounded by the fact that there do exist black holes and black rings with the same mass, spin, *and area*. From this perspective, the black ring is perhaps an unexpected complication.

The map between black holes and string states becomes much more precise if we consider charged BPS states (extremal black holes), which are protected by supersymmetry as we vary the coupling, or even for systems that are slightly excited above the BPS ground state (near-extremal black holes).[2] Therefore, in order to investigate the issues discussed above, we construct in this paper a family of charged black ring solutions that, near the BPS extremal limit, can be identified with configurations of D-branes.

When lifted to six dimensions, black rings become *black tubes*. The BPS ground states of the tubes that appear in our study belong to a class that has received recent attention. They can be referred to, generically, as *supertubes*. Perhaps the best known example is a configuration of D0-branes and fundamental strings (F1) bound to a cylindrical D2-brane tube [7]. Its supergravity description, in different dimensions, was obtained in [8]. From the viewpoint of the worldvolume theory of the D2 brane, the dissolved D0 and F1 appear as crossed magnetic and electric fields. These produce a Poynting vector tangent to the circular sections of the tube, and this gives the system an angular momentum. Other supertubes are obtained from this one via U-duality transformations. For instance, T-duality along the direction of the tube gives a helical D-string, which carries linear momentum as it coils around the tube axis [9]. Further dualizations give a

[2] More than on supersymmetry, the success of the stringy description relies on an AdS_3 structure near the horizon. This is also present in the solutions we construct.

six-dimensional configuration where a D1-D5 bound state rotates on a cylinder. The D1 and the D5 share the longitudinal direction of the tube, with the other four directions of the D5-brane wrapping a four-torus. In the map from the supertube with D0-F1 charges to the one with D1-D5 charges, the circular section of the original D2 tube is mapped into a ring of Kaluza-Klein monopoles, and the longitudinal direction of the tube, which is compactified, corresponds to the U(1) fiber of the KK-monopoles. So the D1 and D5 branes are bound to a KK-monopole tube. A supergravity solution for a BPS D1-D5 supertube was first obtained in [10], and has been generalized and extensively studied in a series of papers [11]–[15] (even if only the last among these refers to them as supertubes).

The D1-D5 bound state is capable of carrying a momentum wave along the D1-D5 intersection. We construct a three-charge black ring, which, in addition to D1 and D5 charges, carries momentum charge (along the sixth-dimensional direction of the tube). However, this extra charge results in pathologies such as closed timelike curves (CTCs) at all points. In fact we find that the boost that generates the momentum is incompatible with the KK-monopole structure in the tube. So the way in which these CTCs appear is different from other recently studied instances of CTCs in rotating systems. Setting one of the charges, e.g., momentum, to zero, eliminates the pathologies, so in order to study the microscopic string description we focus on D1-D5-charged black tubes. The solution with equal D1 and D5 charges is dual to the charged black ring obtained recently in [16] (which can be interpreted as carrying equal F1 and momentum charges).

The D1-D5-charged black tubes can be regarded as thermally excited supertubes. The non-extremal solutions have regular, non-degenerate horizons of finite area and, as in the neutral case, there exist black holes and black rings with the same values of the mass, spin, and charges. By exploiting the detailed knowledge of the microscopics of the D1-D5 system, we find that the stringy descriptions of black holes and black rings are indeed different. This indicates how string theory can, in this case, resolve the duplicity between such objects. We must add, though, that we do not understand yet how string theory distinguishes between the two black rings that have equal asymptotic charges.

The remainder of the paper is organized as follows: In section 2 we introduce the neutral black ring, with some improvements of the original description in [2]. The new three-charge black rings are given in section 3, and their physical parameters are computed in section 4. In section 5 we study further the structure of the three-charge black rings, and find pathologies, including CTCs, from combining a boost with a KK-monopole. Thereafter we set the momentum charge to zero. Section 6 describes the extremal limit of the D1-D5-charged ring, and the connection with supertubes. In section 7 we discuss the microscopic description of spinning black holes and rings with D1-D5 charges, and show that they have a very different description as states of string theory. Section 8 contains the conclusions and outlook of this work. Appendix A provides the intermediate steps leading from the neutral black ring to the three-charge black ring, while appendix B shows how the BMPV black hole [17, 18] can be obtained as an extremal limit of our solutions.

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2. Neutral black ring

We generate the new family of charged black rings by performing a series of transformations on the five-dimensional neutral black ring of [2], so we begin with an extended review of this solution. It is a vacuum solution with metric

$$\begin{aligned} ds^2 &= -\frac{F(x)}{F(y)}\left(dt + R\sqrt{\lambda\nu}(1+y)d\psi\right)^2 + \\ &+\frac{R^2}{(x-y)^2}\left[-F(x)\left(G(y)d\psi^2 + \frac{F(y)}{G(y)}dy^2\right) + F(y)^2\left(\frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right)\right] \end{aligned} \tag{2.1}$$

with

$$F(\xi) = 1 - \lambda\xi\,, \qquad G(\xi) = (1-\xi^2)(1-\nu\xi)\,. \tag{2.2}$$

This form of the solution is slightly different from the original one used in [2, 16]. First, we have made the minor notational change $1/A = R$, since it is more sensible to have a radius scale for the ring rather than the "acceleration parameter" A (which came from the C-metric origin of this solution). Below we will see in what sense R is a radius of the ring. Second, we have adopted the form of the cubic $G(\xi)$ proposed in [19], which yields simple explicit forms for its three roots,

$$\xi_2 = -1\,, \qquad \xi_3 = +1\,, \qquad \xi_4 = \frac{1}{\nu}\,. \tag{2.3}$$

Finally, having the roots of $G(\xi)$ in this form suggests also to rename the root ξ_1 of $F(\xi)$ as $\xi_1 = \lambda^{-1}$. The solution then has dimensionless parameters λ and ν, and a length scale R.

The variables x and y take values in

$$-1 \le x \le 1\,, \qquad -\infty < y \le -1\,, \quad \lambda^{-1} < y < \infty\,. \tag{2.4}$$

As shown in [2], in order to balance forces in the ring one must identify ψ and ϕ with equal period

$$\Delta\phi = \Delta\psi = \frac{4\pi\sqrt{F(-1)}}{|G'(-1)|} = \frac{2\pi\sqrt{1+\lambda}}{1+\nu}\,. \tag{2.5}$$

This eliminates the conical singularities at the fixed-point sets $y = -1$ and $x = -1$ of the Killing vectors ∂_ψ and ∂_ϕ, respectively. There is still the possibility of conical singularities at $x = +1$. These can be avoided in two manners. Fixing

$$\lambda = \lambda_c \equiv \frac{2\nu}{1+\nu^2} \quad \text{(black ring)} \tag{2.6}$$

makes the circular orbits of ∂_ϕ close off smoothly also at $x = +1$. Then (x, ϕ) parametrize a two-sphere, ψ parametrizes a circle, and the solution describes a black ring. Alternatively, if we set

$$\lambda = 1 \quad \text{(black hole)} \tag{2.7}$$

then the orbits of ∂_ϕ do not close at $x = +1$. Then (x, ϕ, ψ) parametrize an S^3 at constant t, y. The solution is the same as the spherical black hole of [1] with a single rotation parameter. Both for black holes and black rings, $|y| = \infty$ is an ergosurface, $y = 1/\nu$ is the event horizon, and the inner, spacelike singularity is reached as $y \to \lambda^{-1}$ from above.

The parameter ν varies in

$$0 \leq \nu < 1\,. \tag{2.8}$$

As $\nu \to 0$ we recover a non-rotating black hole, or a very thin black ring. At the opposite limit, $\nu \to 1$, both the black hole and the black ring get flattened along the plane of rotation, and at $\nu = 1$ result into the same solution with a naked ring singularity.

We will often find it convenient to work with λ as an independent parameter, to be eventually fixed to the equilibrium value λ_c. If we allow for values of λ other than (2.6) or (2.7), then whenever

$$\nu < \lambda < 1 \tag{2.9}$$

we find a black ring solution regular on and outside the horizon, except for a conical singularity on the disk bounded by the inner rim of the ring ($x = +1$). If $\nu < \lambda < \lambda_c$ then the ring is rotating faster than the equilibrium value, and there is a conical deficit balancing the excess centrifugal force. If instead $\lambda_c < \lambda < 1$ then the rotation is too slow and a conical excess appears. If $\lambda \leq \nu$ the horizon is replaced by a naked singularity. We shall refer to the solution with $\lambda = \lambda_c$ as the equilibrium, or balanced black ring. All these qualitative features will remain unchanged for the non-extremal charged rings that we study below.

The mass, spin, area, temperature and angular velocity at the horizon for these solutions are given by[3]

$$\begin{aligned}
M_0 &= \frac{3\pi R^2}{4G}\frac{\lambda(\lambda+1)}{\nu+1}\,, \qquad J_0 = \frac{\pi R^3}{2G}\frac{\sqrt{\lambda\nu}(\lambda+1)^{5/2}}{(1+\nu)^2}\,, && (2.10)\\
\mathcal{A}_0 &= 8\pi^2 R^3 \frac{\lambda^{1/2}(1+\lambda)(\lambda-\nu)^{3/2}}{(1+\nu)^2(1-\nu)}\,, \qquad T_0 = \frac{1}{4\pi R}\frac{1-\nu}{\lambda^{1/2}(\lambda-\nu)^{1/2}}\,, && (2.11)\\
\Omega_0 &= \frac{1}{R}\sqrt{\frac{\nu}{\lambda(1+\lambda)}}\,. && (2.12)
\end{aligned}$$

We use the subscript $_0$ for quantities that refer to the neutral ring, as opposed to the charged rings below. Formulas (2.10), (2.11) and (2.12) are valid in general for λ and ν in the ranges (2.8), (2.9), as long as there is no conical defect at infinity, i.e., when (2.5) is satisfied.

Focusing on the dimensionless quantity

$$\frac{27\pi}{32G}\frac{J_0^2}{M_0^3} = \begin{cases} \dfrac{2\nu}{\nu+1} & \text{(black hole)} \\ \dfrac{(1+\nu)^3}{8\nu} & \text{(balanced black ring)} \end{cases} \tag{2.13}$$

one easily sees that for black holes it grows monotonically from 0 to 1, while for (equilibrium) black rings it is infinite at $\nu = 0$, decreases to a minimum value $27/32$ at $\nu = 1/2$ [16],

[3] The temperature here is defined as the surface gravity of the horizon divided by 2π. We prefer not to use the euclidean approach, because there is no analytic continuation of the black ring that produces a non-singular, real euclidean solution.

and then grows to 1 at $\nu = 1$. This implies that in the range

$$\frac{27}{32} \leq \frac{27\pi}{32G} \frac{J_0^2}{M_0^3} < 1 \tag{2.14}$$

there exist one black hole and two black rings with the same value of the spin for fixed mass. This regime of non-uniqueness occurs when the parameter ν takes values in

$$\sqrt{5} - 2 \leq \nu < 1 \tag{2.15}$$

for black rings, and in

$$\frac{27}{37} \leq \nu < 1 \tag{2.16}$$

for black holes. Bear in mind that when we speak about non-uniqueness we always refer to equilibrium black rings.

There is a limit in which many features of the black ring become particularly clear. Take λ and ν as independent parameters, and let both of them approach zero at the same rate (so ν/λ remains finite). Then we obtain a ring with a large spin for a given mass. The ring is very thin ('hula-hoop'-like), and locally it approaches the geometry of a boosted black string. To recover this limit, we focus on a region near the horizon, of size small compared to R, and scale

$$R \to \infty\,, \qquad \lambda, \nu \to 0 \tag{2.17}$$

while keeping νR and λR finite. Define new parameters r_0, σ, and coordinates r, θ, w that remain finite in the limit,

$$\begin{aligned} \nu R &= r_0 \sinh^2 \sigma\,, \qquad & \lambda R &= r_0 \cosh^2 \sigma\,, \\ r &= -R\frac{F(y)}{y}\,, \qquad & \cos\theta &= x\,, \qquad w = R\psi\,, \end{aligned} \tag{2.18}$$

(so $|y|$ is large) to find that the black ring (2.1) goes over to

$$ds^2 = -\bar{f}\left(dt - \frac{r_0 \sinh 2\sigma}{2r\bar{f}} dw\right)^2 + \frac{f}{\bar{f}} dw^2 + \frac{dr^2}{f} + r^2 d\Omega_2^2\,, \tag{2.19}$$

with

$$f = 1 - \frac{r_0}{r}\,, \qquad \bar{f} = 1 - \frac{r_0 \cosh^2 \sigma}{r}\,. \tag{2.20}$$

This a boosted black string, with boost parameter σ. When $\lambda \to \nu$ the boost becomes light-like. For the equilibrium ring we require instead $\lambda = \lambda_c \to 2\nu$, and the boost is $\sinh|\sigma| = 1$. This happens to be a rather special value. The ADM stress-energy tensor (see e.g., [20]) of the boosted black string is

$$\begin{aligned} T_{tt} &= \frac{r_0}{4G}(1 + \cosh^2 \sigma)\,, \\ T_{tw} &= \frac{r_0}{4G} \sinh\sigma \cosh\sigma\,, \\ T_{ww} &= \frac{r_0}{4G}(\sinh^2 \sigma - 1)\,. \end{aligned} \tag{2.21}$$

Here T_{tt} and T_{tw} are the energy and momentum linear densities of the black string, while T_{ww} is its pressure density. The limiting boost of the balanced black ring is $\sinh|\sigma| = 1$, so its limit is a pressureless black string. This absence of pressure reflects the delicate balance of forces that the black ring represents. Furthermore, one can check that for the balanced black ring the limiting mass and spin are such that

$$\frac{M_0}{2\pi R} \to T_{tt}\,, \qquad \frac{J_0}{2\pi R^2} \to T_{tw}\,. \tag{2.22}$$

Since T_{tt} and T_{tw} are densities of mass and momentum per unit length, we see that R must be interpreted as the circle radius of very thin rings, in the sense that, in the limit, the coordinate w is identified as $w \sim w + 2\pi R$. So $2\pi R$ is the proper length measured at a large transverse distance from the string. This is not the same as the proper length at the horizon, which is $\Delta\psi\,\sqrt{g_{\psi\psi}}|_{y=1/\nu} \to 2\pi R\sqrt{g_{ww}}|_{r=r_0} = 2\pi R\cosh\sigma$, and is expanded by the pressure of the boost. Consistently with this interpretation, the area per unit length of a thin large ring is $\mathcal{A}_0/2\pi R \to 4\pi r_0^2\cosh\sigma$, i.e., it becomes equal to the area per unit length of the boosted black string. Also, in this limit the angular velocity (2.12) is $\Omega_0 \to R^{-1}\tanh\sigma$, so $\Omega_0 R$ becomes the boost velocity.

Eq. (2.22) for the mass assumes that the ring is balanced, i.e., pressureless. For unbalanced rings the mass gets an additional contribution from the pressure of the conical defect disk.

3. Charging up the ring

The method to build the three-charge black ring does not differ in essence from the one that gave the three-charge rotating black hole in five dimensions [21, 22]. The final solution is given below in eqs. (3.5)–(3.14), for the case where the charges correspond to fundamental strings (F1), NS5-branes and momentum P. Here and in appendix A we provide an outline and some of the intermediate steps leading to it.

A rotating charged black ring with a single electric charge was constructed in [16] as a solution to low energy heterotic string theory. The charged solution was generated from the neutral solution (2.1) by application of a Hassan-Sen transformation [23]. This involves a boost in an internal direction, and the first step in generalizing the charged black ring of [16] is to realize that such a boost can be imitated by Lorentz boosts and T-duality in an extra spatial direction followed by a Kaluza-Klein reduction to five dimensions. We elaborate on this in the following as we derive a solution for a black ring with two charges.

Add to the metric (2.1) a flat sixth dimension z. We also add four flat directions x^i, $i = 6, 7, 8, 9$, wrapped on a T^4, but these will play a more passive role. A Lorentz boost, $dt \to \cosh\alpha_5\, dt + \sinh\alpha_5\, dz$ and $dz \to \sinh\alpha_5\, dt + \cosh\alpha_5\, dz$, gives the solution linear momentum in the z-direction, and subsequent T-dualization of the z-direction exchanges the momentum with a fundamental string charge from the 3-form flux [24]. The choice of subscripts in the boosts α_i will become clearer later. We can then apply another Lorentz boost α_1 in the z-direction to get a solution with both charge and momentum. The resulting six-dimensional black ring (or rather black tube) solution is given in appendix A in eqs.

(A.1)–(A.5). It is a solution to the classical equations of motion obtained from the action of the low energy NS-NS sector of the superstring compactified on T^4,

$$S_6 = \frac{1}{2\kappa_6^2}\int d^6x\,\sqrt{-\hat{g}}\,e^{-2\Phi}\left(R^{(6)} + 4(\nabla\Phi)^2 - \frac{1}{12}\hat{H}^2\right). \tag{3.1}$$

Compactify the z-direction on a circle of radius R_z. A Kaluza-Klein reduction along the z-direction, using the ansatz

$$\hat{g}_{MN}dx^M dx^N = g_{\mu\nu}dx^\mu dx^\nu + e^{2\sigma}(dz + A_\mu^{(n)}dx^\mu)^2\,, \tag{3.2}$$

where $x^M = (x^\mu, z)$ and $e^{2\sigma} = g_{zz}$, leads to a five-dimensional action

$$\begin{aligned} S_5 = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}e^{-2\Phi+\sigma}\Bigg(&R^{(5)} + 4(\nabla\Phi)^2 - 4\nabla\Phi\nabla\sigma - \frac{1}{12}H^2 - \\ &- \frac{1}{4}e^{2\sigma}(F^{(n)})^2 - \frac{1}{4}e^{-2\sigma}(F^{(1)})^2\Bigg)\,, \end{aligned} \tag{3.3}$$

with $\kappa_5^2 = \kappa_6^2/(2\pi R_z)$. In (3.3), $F_{\mu\nu}^{(n)} = \partial_\mu A_\nu^{(n)} - \partial_\nu A_\mu^{(n)}$, and we have defined $A_\mu^{(1)} = \hat{B}_{\mu z}$, so that $F_{\mu\nu}^{(1)} = \partial_\mu A_\nu^{(1)} - \partial_\nu A_\mu^{(1)} = \hat{H}_{z\mu\nu}$. The 3-form flux involves a Chern-Simons term and is given by

$$H_{\mu\nu\rho} = (\partial_\mu \hat{B}_{\nu\rho} - A_\mu^{(n)}F_{\nu\rho}^{(1)}) + \text{cyclic permutations}\,. \tag{3.4}$$

By setting $\sigma = 0$ and $F^{(1)} = F^{(n)}$ one obtains a consistent truncation of the theory (3.3) and the resulting action is that of the heterotic string in five dimensions with only one U(1) subgroup included (see also [25]). Defining the effective dilaton $\Phi_{\text{eff}} = \Phi - \sigma/2$, the Einstein frame metric is $g_{\mu\nu}^{\text{E}} = e^{-\frac{4}{3}\Phi_{\text{eff}}}g_{\mu\nu}$

The five-dimensional solution obtained by KK reducing the solution (A.1)–(A.5) describes a rotating black ring with two charges, one from the KK gauge field $A_\mu^{(\text{n})}$ and the other from the gauge field $A_\mu^{(1)}$. Including in the metric (A.1) the four flat directions x^i, $i = 6, 7, 8, 9$, we can view it as a solution with fundamental string charge $F1(z)$ (corresponding to α_5) and momentum in the z-direction $P(z)$ (from the boost α_1).

For the special case where the boost parameters are $\alpha_5 = \alpha_1$, the gauge fields become identical and $\sigma = 0$, so this is a solution of the low energy heterotic string action. Up to normalizations of the gauge fields, this is exactly the solution with one charge found in [16].

We now proceed to find a solution with three charges. We continue from the solution (A.1), plus the four flat directions x^i, viewed as a type-IIB solution with P and F1 charges. A sequence of dualities maps [12, 15] this solution to a supergravity solution describing the D1-D5-system. Boosting the resulting system in the direction z before KK reducing will then give us a black ring with three charges. We describe the procedure briefly in this section and leave the details for appendix A.

S-dualizing takes the black tube solution with [$P(z)$, F1(z)] charges to a solution with charges [$P(z)$, D1(z)]. T-dualizing the four directions x^i, then gives charges [$P(z)$, D5(z6789)], and S-dualizing again takes us to a [$P(z)$, NS5(z6789)] system. Now T-dualizing z converts the momentum into F1 charge, giving a solution [F1(z), NS5(z6789)]

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of type-IIA. A T-dualization in any one of the x^i direction is trivial and takes us to type-IIB, and we can then S-dualize the solution to get a tube solution with [D1(z), D5(z6789)] charges.

For either of the solutions [F1(z), NS5(z6789)] or [D1(z), D5(z6789)] we can apply a boost with parameter α_n in the z-direction and then KK reduce to five dimensions. In appendix A, we give the metric and fields for the [$P(z)$, F1(z), NS5(z6789)] solution. Reducing (A.10) to five dimensions we find a black ring solution of (3.3) with Einstein frame metric

$$\begin{aligned} ds_5^2 = & -\frac{1}{(h_5 h_1 h_n)^{2/3}} \frac{F(x)}{F(y)} \Big(dt - \sqrt{\lambda\nu} R(1+y) \cosh\alpha_5 \cosh\alpha_1 \cosh\alpha_n \, d\psi + \\ & \qquad + \sqrt{\lambda\nu} R(1+x) \sinh\alpha_5 \sinh\alpha_1 \sinh\alpha_n \, d\phi \Big)^2 \\ & + (h_5 h_1 h_n)^{1/3} \frac{R^2}{(x-y)^2} \Bigg[-F(x) \left(G(y) d\psi^2 + \frac{F(y)}{G(y)} dy^2 \right) + \\ & \qquad + F(y)^2 \left(\frac{dx^2}{G(x)} + \frac{G(x)}{F(x)} d\phi^2 \right) \Bigg] , \end{aligned} \tag{3.5}$$

where we have defined

$$\begin{aligned} h_i(x,y) \equiv & \frac{F(y)\cosh^2\alpha_i - F(x)\sinh^2\alpha_i}{F(y)} \\ = & 1 + \frac{\lambda(x-y)}{F(y)} \sinh^2\alpha_i \, . \end{aligned} \tag{3.6}$$

The dilaton Φ and the extra scalar σ are given by

$$e^{-2\Phi} = \frac{h_1(x,y)}{h_5(x,y)} , \qquad e^{2\sigma} = \frac{h_n(x,y)}{h_1(x,y)} . \tag{3.7}$$

The gauge fields are

$$A_t^{(1)} = \frac{(x-y)\lambda \sinh 2\alpha_1}{2F(y) h_1(x,y)} \tag{3.8}$$

$$A_\psi^{(1)} = \frac{\sqrt{\lambda\nu} R(1+y) F(x) \cosh\alpha_5 \sinh\alpha_1 \cosh\alpha_n}{F(y) h_1(x,y)} \tag{3.9}$$

$$A_\phi^{(1)} = -\frac{\sqrt{\lambda\nu} R(1+x) \sinh\alpha_5 \cosh\alpha_1 \sinh\alpha_n}{h_1(x,y)} , \tag{3.10}$$

and

$$A_\mu^{(n)} = A_\mu^{(1)}[\alpha_1 \leftrightarrow \alpha_n] . \tag{3.11}$$

There is an antisymmetric tensor field with components

$$B_{t\psi} = -\frac{\sqrt{\lambda\nu} R(1+y) F(x) \cosh\alpha_5 \sinh\alpha_1 \sinh\alpha_n}{F(y) h_1(x,y)} \tag{3.12}$$

$$B_{t\phi} = \frac{\sqrt{\lambda\nu} R(1+x) \sinh\alpha_5 \cosh\alpha_1 \cosh\alpha_n}{h_1(x,y)} \tag{3.13}$$

$$B_{\psi\phi} = -\frac{1}{2}\lambda R^2 \sinh 2\alpha_5 \left(\frac{G(x)}{x-y} + k(x) + \frac{\nu F(x)(1+x)(1+y)\sinh^2\alpha_1}{F(y) h_1(x,y)} \right) , \tag{3.14}$$

where $k(x) = x(1+\nu) - \nu x^2 + \text{const}$. In five dimensions, the 3-form flux H given in (3.4) is dual to a 2-form field strength $F^{(5)} = e^{-2\Phi_{\rm eff}} \star H$. $F^{(5)}$ can be obtained from the gauge potential

$$A^{(5)}_\mu = A^{(1)}_\mu[\alpha_1 \leftrightarrow \alpha_5]\,. \tag{3.15}$$

Each boost is associated to one kind of charge,

$$\begin{aligned} \alpha_5 &\longrightarrow \mathrm{NS5}\,, \\ \alpha_1 &\longrightarrow \mathrm{F1}\,, \\ \alpha_n &\longrightarrow P\,. \end{aligned} \tag{3.16}$$

Under S-duality, the NS5 and F1 transform into D5 and D1. Setting any one of the charges to zero, we recover the two-charge black ring. Originally, this was obtained by a reduction of the [$P(z)$, F1(z)]-system, but the three charges [P, F1, NS5] can be permuted by U-duality transformations of the ten-dimensional solution. The Einstein-frame metric (3.5) is invariant under such transformations, however, the radii of compactification and the dilaton do change. If we regard the [$P(z)$, F1(z), NS5($z6789$)]-ring as a solution to type-IIA supergravity, then it can be lifted to a solution of 11D supergravity, with an additional coordinate ζ, of the form [$P(z)$, M2($z\zeta$), M5($z6789$)]. Reduction along the coordinate z yields another type-IIA solution, with D0, F1 and D4 charges, which is also T-dual to the D1-P-D5 solution.

The particular case of the solution (3.5) where all charges are equal, $\alpha_5 = \alpha_1 = \alpha_n$, yields a solution to minimal $N = 2$ supergravity in five dimensions.[4]

One remarkable feature of the new solution (3.5) is that, with the inclusion of the third charge the black ring has acquired angular momentum along a second independent axis, the direction ϕ of the 2-sphere. We will see, however, that this is more a problem than a virtue.

4. Physical parameters

Many of the qualitative properties of non-extremal black rings are the same as for neutral black rings — a main distinguishing feature will be discussed in the next section. The parameters ν and λ, and the coordinates x and y, vary in the same ranges as in section 2, and ϕ and ψ are identified with the same periods (2.5). Again, the event horizon is located at $y = 1/\nu$, and $y > 1/\nu$ defines the ergoregion.

If we make the choice $\lambda = 1$ of the spherical black hole class of solutions (see (2.7)), we recover a particular case of the three-charge rotating black hole in five dimensions of [21, 22].[5] The change from the coordinates used in this paper (appropriate for the ring) to the more conventional Boyer-Lindquist-type coordinates, can be found in [2]. When all charges are nonzero this solution has a smooth inner horizon at $y = 1/\lambda = 1$, but if either one of the charges is set to zero, the inner horizon becomes a curvature singularity.

[4] This solution was obtained previously by E. Teo, and then discarded due to the pathological CTCs to be described in section 5 [26].

[5] The solutions in [21, 22] have an additional parameter corresponding to the second angular momentum of the seed neutral Myers-Perry black hole.

When $\nu < \lambda < 1$, and up to an important issue to be discussed in next section, the horizon is topologically $S^2 \times S^1$ and the solution is a spinning black ring, which is balanced and free of conical singularities only when $\lambda = \lambda_c$ (2.6). For the ring there is no smooth inner horizon; behind the event horizon at $y = 1/\nu$ the curvature blows up at $y = 1/\lambda$.

The physical quantities for the charged black ring are computed in the Einstein frame. The ADM mass and angular momenta are

$$\begin{aligned} M &= \frac{1}{3} M_0 \left(\cosh 2\alpha_5 + \cosh 2\alpha_1 + \cosh 2\alpha_n \right) , & (4.1) \\ J_\psi &= J_0 \cosh \alpha_5 \cosh \alpha_1 \cosh \alpha_n \, , & (4.2) \\ J_\phi &= -J_0 \sinh \alpha_5 \sinh \alpha_1 \sinh \alpha_n \, , & (4.3) \end{aligned}$$

expressed in terms of the mass M_0 and the angular momentum J_0 of the neutral solution (2.10).

The ring couples electrically to the gauge fields and the three corresponding charges are computed as

$$Q_i = \frac{1}{2\Omega_3} \int_{S^3 \text{ at } \infty} e^{-2\Phi_i} \star F^{(i)} \, , \tag{4.4}$$

where $\Omega_3 = 2\pi^2$ is the area of a unit three-sphere, $F^{(i)}$ is a field strength, $F^{(i)} = dA^{(i)}$, and $\Phi_1 = \Phi + \sigma/2$, $\Phi_5 = -\Phi_{\text{eff}}$, and $\Phi_n = \Phi - 3\sigma/2$, as can be seen from the action (3.3). We find

$$Q_5 = \frac{4G}{3\pi} M_0 \sinh 2\alpha_5 \, , \qquad Q_1 = \frac{4G}{3\pi} M_0 \sinh 2\alpha_1 \, , \qquad Q_n = \frac{4G}{3\pi} M_0 \sinh 2\alpha_n \, . \tag{4.5}$$

With this definition the six-dimensional momentum is $P = \frac{\pi}{4G} Q_n$. Note that for all values of the parameters, the charges and the mass satisfy the inequality

$$|Q_1| + |Q_5| + |Q_n| \leq \frac{4G}{\pi} M \, , \tag{4.6}$$

generalizing the bound found in [16].

It is clear that, again, there is a range of parameters where there exist a black hole and two black rings with the same values of the mass, spin, and the three charges Q_i. This happens for parameters in the ranges (2.14), (2.15), (2.16).

The area of the horizon and its temperature admit also simple expressions in terms of their values (2.11) for the neutral solution,

$$\begin{aligned} \mathcal{A} &= \mathcal{A}_0 \cosh \alpha_5 \cosh \alpha_1 \cosh \alpha_n \, , & (4.7) \\ T &= \frac{T_0}{\cosh \alpha_5 \cosh \alpha_1 \cosh \alpha_n} \, . & (4.8) \end{aligned}$$

The charges of the black ring are proportional to the total number of branes and momentum quanta. To obtain these numbers, first we choose units where $\alpha' = 1$, and take for simplicity the volume of the compact four dimensions x^i to be $(2\pi)^4$, so the five-dimensional Newton's constant is

$$G = \frac{\pi g_s^2}{4 R_z} \, , \tag{4.9}$$

where R_z is the radius of the direction z along the string/fivebrane intersection, and g_s is the string coupling constant. Since the solution is not extremal, there can be both branes and antibranes, as well as left and right movers. The integer charges measure the difference between their numbers. For the D1-D5-P black ring, the integer-quantized total numbers of D1-branes, D5-branes, and momentum units are

$$n_1 - \bar{n}_1 = g_s^{-1} Q_1 \,, \qquad n_5 - \bar{n}_5 = g_s^{-1} Q_5 \,, \qquad n_L - n_R = n = R_z P = \frac{R_z^2}{g_s^2} Q_n \,. \tag{4.10}$$

For the F1-NS5-P ring the integer charges of F1 and NS5 are $g_s^{-2} Q_1$, and Q_5, respectively.

The black ring also carries other local (dipole-type) charges. The full structure of the solution will be uncovered in the next section.

In the same manner as we did for thin neutral rings, we can blow up the region near the ring by taking the limits (2.17), (2.18). This yields a charged black string in five dimensions, or a charged black 2-brane if lifted to six dimensions. If we take the limit for the F1-NS5-P six-dimensional black tube (A.10), then the tube is straightened out to a 2-brane spanning the coordinates z, w,

$$\begin{aligned} ds^2 \;=\; & \frac{h_n}{h_1}\Bigg(dz + \frac{r_0 \cosh^2\sigma\, \sinh 2\alpha_n}{2 r h_n}\, dt - \frac{r_0 \sinh 2\sigma \cosh\alpha_5 \cosh\alpha_1 \sinh\alpha_n}{2 r h_n}\, dw + \\ & \qquad + \frac{r_0 \sinh 2\sigma \sinh\alpha_5 \sinh\alpha_1 \cosh\alpha_n}{2 h_n} (\cos\theta + 1)\, d\phi\Bigg)^2 - \\ & -\frac{\bar{f}}{h_1 h_n}\Bigg(dt + \frac{r_0 \sinh 2\sigma}{2 r \bar{f}} \cosh\alpha_5 \cosh\alpha_1 \cosh\alpha_n\, dw + \\ & \qquad + \frac{r_0 \sinh 2\sigma}{2} \sinh\alpha_5 \sinh\alpha_1 \sinh\alpha_n\, (\cos\theta + 1)\, d\phi\Bigg)^2 \\ & + h_5 \left(\frac{f}{\bar{f}} dw^2 + \frac{dr^2}{f} + r^2 d\Omega_2^2\right) . \end{aligned} \tag{4.11}$$

Here

$$h_i = 1 + \frac{r_0}{r} \cosh^2\sigma\, \sinh^2\alpha_i \,. \tag{4.12}$$

Observe that we have left the boost parameters α_i fixed, so this is not an extremal, nor near-extremal, limit. The charges Q_i have been simply scaled up, like the mass, as R, so as to keep a finite charge per unit length of the ring.[6]

A calculation of the ADM stress-energy tensor as in (2.21), yields again vanishing pressure for the limit of the balanced ring. The mass, momentum and charge densities agree, as in (2.22), with those for a string of (asymptotic) length R. The solution (4.11) can also be obtained by direct application of the transformations of section 3 to the boosted black string.

[6] Close to extremality, this is different than the near-horizon limit of the near-extremal black ring. The details will be discussed elsewhere.

5. Closed timelike curves, and Kaluza-Klein-monopole tube

When all three charges are non-zero the black ring metric (3.5) contains the one-form

$$\mathcal{C} \equiv dt - \sqrt{\lambda\nu}R(1+y)\cosh\alpha_5\cosh\alpha_1\cosh\alpha_n\, d\psi + \sqrt{\lambda\nu}R(1+x)\sinh\alpha_5\sinh\alpha_1\sinh\alpha_n\, d\phi \tag{5.1}$$

with the effect that the orbits of ∂_t are non-trivially fibered over the $S^2 \times S^1$ surfaces parametrized by (x, ψ, ϕ). We should be careful to avoid Dirac-Misner string singularities[7] in the geometry, which may arise at the fixed-points of the orbits of ∂_ψ and ∂_ϕ. Since $\mathcal{C}_\psi$ vanishes at the only possible fixed-point set of ∂_ψ, $y = -1$, there is no singularity there. However, if (x, ϕ) is to describe a two-sphere then ∂_ϕ will have fixed-points at $x = \pm 1$. Now, $\mathcal{C}_\phi = 0$ at $x = -1$, but not at $x = +1$. We can remove the Dirac-Misner singularity at $x = +1$ in a standard manner, taking a different coordinate patch to cover this region. For some $\epsilon > 0$, take t as the coordinate in the patch $-1 \le x < \epsilon$, and t' in $-\epsilon < x \le 1$. In the overlap region $|x| < \epsilon$, relate them via

$$t' = t + (2\sqrt{\lambda\nu}\, R\sinh\alpha_5\sinh\alpha_1\sinh\alpha_n)\,\phi\,. \tag{5.2}$$

With this construction, the Dirac-Misner singularities are absent from both poles $x = \pm 1$. But we have to pay a price: the matching (5.2) requires that t (and t') be identified with periodicity

$$\Delta t = (2\sqrt{\lambda\nu}\, R\sinh\alpha_5\sinh\alpha_1\sinh\alpha_n)\,\Delta\phi \tag{5.3}$$

(or an integer fraction of this). This introduces closed timelike curves outside the horizon, even at infinity. So, either we have string singularities, or we introduce naked CTCs: The solution with three non-zero charges is pathological. This problem is instead absent from the charged black hole solution, where ∂_ϕ has a fixed-point set only at $x = -1$.

To further clarify the origin of the problem, let us set the momentum charge to zero, $\alpha_n = 0$, so the pathology disappears, and consider the solution lifted to six dimensions, as in (A.10). The relevant structure of the solution is the same whether we consider the F1-NS5 or D1-D5 black tubes. For later convenience we give the full ten-dimensional D1-D5 solution, with (string frame) metric

$$\begin{aligned} ds^2 \;=\; & \frac{1}{\sqrt{h_1 h_5}}\left(dz - \sqrt{\lambda\nu}R(1+x)\sinh\alpha_5\sinh\alpha_1\, d\phi\right)^2 - \\ & -\frac{1}{\sqrt{h_1 h_5}}\frac{F(x)}{F(y)}\left(dt - \sqrt{\lambda\nu}R(1+y)\cosh\alpha_5\cosh\alpha_1\, d\psi\right)^2 + \\ & +\sqrt{h_1 h_5}\frac{R^2}{(x-y)^2}\left[-F(x)\left(G(y)d\psi^2 + \frac{F(y)}{G(y)}dy^2\right) + F(y)^2\left(\frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right)\right] + \\ & +\sqrt{\frac{h_1}{h_5}}\sum_{i=6}^{9} dx^i dx^i\,, \end{aligned} \tag{5.4}$$

[7] A Dirac-Misner string is a gravitational analogue of the Dirac string of gauge fields, first discussed by Misner in [27] for the Taub-NUT solution.

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dilaton

$$e^{-2\Phi} = \frac{h_5(x,y)}{h_1(x,y)}\,, \tag{5.5}$$

and the nonzero components of the RR 2-form are

$$\begin{aligned}
C_{t\phi} &= -\frac{\sqrt{\lambda\nu}R(1+x)\sinh\alpha_5\cosh\alpha_1}{h_1(x,y)} && (5.6)\\
C_{\psi\phi} &= \frac{1}{2}R^2\lambda\sinh 2\alpha_5\left(\frac{G(x)}{x-y}+k(x)+\frac{\nu F(x)(1+x)(1+y)\sinh^2\alpha_1}{F(y)h_1(x,y)}\right) && (5.7)\\
C_{tz} &= -\frac{(x-y)\lambda\sinh 2\alpha_1}{2F(y)h_1(x,y)} && (5.8)\\
C_{\psi z} &= -\frac{\sqrt{\lambda\nu}R(1+y)F(x)\cosh\alpha_5\sinh\alpha_1}{F(y)h_1(x,y)}\,. && (5.9)
\end{aligned}$$

Now we see that the direction z is non-trivially fibered over the (x,ϕ) two-sphere. This fibration is actually quite familiar: it is a Hopf fibration due to the presence of a KK-monopole. The KK-monopole extends along the ψ direction (and also along the four compact x^i), so we have a KK-monopole tube. In fact this is not unexpected. The D1-D5 black tube, can be transformed via U-dualities into a D0-F1 black tube. As will become clearer later, the extremal limit of all these solutions are supertubes. It is known that the D0-branes and F-strings in a supertube configuration live in the worldvolume of a cylindrical D2-brane. Mapping this D2 cylinder back into the D1-D5 tube, one obtains a KK-monopole along the ψ circle with KK-direction z. This is the structure we have identified directly above. The KK monopoles in the D1-D5 supertube have also been noted in [15]. They are also present in the non-extremal solutions, as evidenced by the fibration, even if a horizon appears before the 'nut' is reached. Note that the KK-monopoles are absent when we consider spherical black hole solutions.

As before, removal of Dirac-Misner strings from the fibration requires that we identify z with period

$$\Delta z \equiv 2\pi R_z = \frac{2\sqrt{\lambda\nu}\,R\sinh\alpha_5\sinh\alpha_1}{n_{\rm KK}}\,\Delta\phi\,, \tag{5.10}$$

where $n_{\rm KK}$ is an integer that counts the number of KK-monopoles that make up the tube (or the number of times the KK-monopole winds around the tube). Since the KK-monopole is distributed on a circle, its charge is not one of the conserved charges of the black ring. Instead, it appears in the asymptotic region as a magnetic dipole of the field $A^{(n)}$. The local charge can be measured by integrating the KK magnetic flux across an S^2 that intersects the ring at a single point, analogous to the calculation of the local D2 charge for the supertube in [8].

It is actually the combination of the KK-monopole and the boost along the KK-direction z that is responsible for the pathological Dirac-Misner strings that we have found. To see this, consider the solution for a KK-monopole in five dimensions,

$$ds^2 = -dt^2 + H^{-1}\left[dz + q(\cos\theta+1)\,d\phi\right]^2 + H(dr^2 + r^2 d\Omega_2^2) \tag{5.11}$$

with $H = 1 + \frac{q}{r}$. The string at $\theta = 0$ can be removed with adequately chosen coordinate patches. Now boost it along the direction z with boost parameter α_n, to find

$$
\begin{aligned}
ds^2 = & \frac{H_n}{H}\left(dz - \frac{q\sinh 2\alpha_n}{2rH_n}\,dt + \frac{q\cosh\alpha_n}{H_n}(\cos\theta + 1)\,d\phi\right)^2 - \\
& - H_n^{-1}\left(dt - q\sinh\alpha_n(\cos\theta+1)\,d\phi\right)^2 + H(dr^2 + r^2 d\Omega_2^2)
\end{aligned} \tag{5.12}
$$

where $H_n = 1 - \frac{q\sinh^2\alpha_n}{r}$. The essence of the fiber structure of z and t over the S^2 in this solution can be recognized in (4.11), and in general it is clear how the Dirac-Misner strings at the poles of the S^2 force the periodicity of the time t in the same manner as we found for the three-charge black ring. As a matter of fact, the KK-monopole fibration imposes identifications in the geometry that are incompatible with the boost. So besides the CTCs, the geometry (5.12) does not even describe a consistent fibration. This same problem is present for the three-charge black tube.

The dual tube with D0, F1 and D4 charges (the D4 wraps the four internal directions x^i) exhibits the same inconsistencies even if, as a solution to IIA supergravity, it does not contain a KK-monopole. When lifted to eleven dimensions, this system is described by M2-branes and M5-branes on a tube, carrying momentum along a common intersection. Exactly the same solution is obtained by lifting the type-IIA ring with F1, NS5 and momentum charges. This implies that the M-branes are again in the background of a KK-monopole tube, which is incompatible with the momentum wave.

Note that these problems are of a different nature, and more serious, than the CTCs found for the over-rotating charged black hole in five dimensions, which appear as a result of the identifications that compactify the solution from six to five dimensions [28]. Undoing these identifications and going to the universal covering then removes the CTCs. In our case, the presence of the KK-monopole in the D1-D5 or F1-NS5 black tube *forces* the direction z to be compact — n_{KK} cannot be zero — and also forbids the possibility of boosting the solution.

So three-charge black rings appear to be unphysical. However, the two-charge solutions obtained by setting any one of the charges to zero are perfectly sensible: the D1-D5 solution does not have momentum, and the D1-P and D5-P do not have a KK-monopole tube. The three-charge solution allows us to obtain any of these two-charge solutions immediately, and is at least a useful way of encoding all these U-dual solutions.

6. Extremal limit

An extremal limit of the solution is obtained by sending at least one of the boost parameters α_i to ∞, while keeping the metric and the corresponding charges Q_i finite. From (4.5) we see that we must send $M_0 \to 0$ while the products $M_0 e^{2\alpha_i}$ remain finite. One possibility is to take $R \to 0$, with λ and ν finite. We show in appendix B that in this way one always recovers the extremal three-charge spherical black hole.

Here we are more interested in the limit where $\lambda, \nu \to 0$ while R stays finite, which preserves the ring-like structure of the solution. Since $\lambda \neq 1$, the possibility that this limit be taken for spherical black holes is excluded. Also, it is possible to have an extremal

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limit for the three-charge ring if λ and ν go to zero at a different rate. But the extremal three-charge solution that results presents the same pathologies that we found for generic three-charge rings, and therefore we will not consider it further. In the following we set the momentum charge to zero. It is easy to redo the analysis when, instead, any of the other two charges is set to zero.

To obtain this extremal limit for the D1-D5 black tube, send $\alpha_1, \alpha_5 \to \infty$ and $\lambda, \nu \to 0$ while keeping

$$\begin{aligned} \lambda e^{2\alpha_i} &= \frac{2Q_i}{R^2}\,, \\ \sqrt{\lambda\nu}e^{\alpha_5+\alpha_1} &= \frac{8G}{\pi}\frac{J}{R^3} = \frac{2g_s^2}{R_z}\frac{J}{R^3}\,, \end{aligned} \tag{6.1}$$

fixed. Note in the last line we have used (4.9).

The extremal solution can be put in a convenient form with a change to new coordinates $(x,\, y) \to (r,\, \theta)$,

$$r^2 = R^2\frac{1-x}{x-y}\,, \qquad \cos^2\theta = \frac{1+x}{x-y}\,. \tag{6.2}$$

Then the metric, written down as a ten-dimensional D1-D5 solution in string frame, is

$$\begin{aligned} ds^2 = &-\frac{1}{\sqrt{h_1 h_5}}\left(dt + \frac{g_s^2}{R_z}\frac{J\sin^2\theta}{\Sigma}\,d\psi\right)^2 + \frac{1}{\sqrt{h_1 h_5}}\left(dz - \frac{g_s^2}{R_z}\frac{J\cos^2\theta}{\Sigma}\,d\phi\right)^2 + \\ &+\sqrt{h_1 h_5}\left[\Sigma\left(\frac{dr^2}{r^2+R^2} + d\theta^2\right) + (r^2+R^2)\sin^2\theta d\psi^2 + r^2\cos^2\theta d\phi^2\right] + \\ &+\sqrt{\frac{h_1}{h_5}}\sum_{i=6}^{9} dx^i dx^i\,. \end{aligned} \tag{6.3}$$

with

$$h_i = 1 + \frac{Q_i}{\Sigma}\,, \qquad \Sigma \equiv r^2 + R^2\cos^2\theta\,. \tag{6.4}$$

When this extremal solution is obtained as a limit of balanced black rings, λ is not a free parameter, instead $\lambda \to 2\nu$ in the limit, so from (6.1) we find that the spin of the extremal ring is fixed to be

$$\begin{aligned} J^2 &= \frac{1}{2}\frac{R_z^2}{g_s^4}Q_1 Q_5 R^2 \\ &= \frac{1}{2}\frac{R_z^2}{g_s^2}n_1 n_5 R^2\,, \end{aligned} \tag{6.5}$$

where we use (4.10) and the fact that for the extremal solution there are no anti-D1 or anti-D5 branes. As described in section 5, the D1 and D5 branes are rotating on top of a KK-monopole tube, which imposes the periodicity condition (5.10) on the z coordinate. In the extremal limit (6.1), equation (5.10) results in

$$n_{\rm KK}R^2 = \frac{g_s^2}{R_z^2}J\,. \tag{6.6}$$

Together with (4.10), this allows us to write the equation (6.5) for the spin of the extremal ring in terms of the numbers of strings, five-branes, and KK-monopoles, as

$$J = \frac{1}{2}\frac{n_1 n_5}{n_{\mathrm{KK}}}\,. \tag{6.7}$$

This two-charge extremal ring, or better, extremal tube in 6D, is in fact a D1-D5 *supertube*, within the class of 1/4-supersymmetric solutions studied in [14, 15]. Another coordinate form, that allows to make contact with the (six-dimensional) supertube solutions in [8], is obtained by changing $(r,\theta) \to (\rho_1, \rho_2)$,

$$\rho_1 = \sqrt{r^2+R^2}\sin\theta\,, \qquad \rho_2 = r\cos\theta\,. \tag{6.8}$$

Then $h_{1,5}$ take the same form as in (6.4), with

$$\Sigma = \sqrt{(\rho_1^2+\rho_2^2+R^2)^2 - 4R^2\rho_1^2}\,, \tag{6.9}$$

and the metric is

$$\begin{aligned} ds^2 =& -\frac{1}{\sqrt{h_1 h_5}}\left(dt + \frac{4G}{\pi}\frac{2J\rho_1^2}{\Sigma(\rho_2^2+R^2+\Sigma+\rho_1^2)}\,d\psi\right)^2 + \\ &+\frac{1}{\sqrt{h_1 h_5}}\left(dz - \frac{4G}{\pi}\frac{J(\rho_2^2+R^2+\Sigma-\rho_1^2)}{\Sigma(\rho_2^2+R^2+\Sigma+\rho_1^2)}\,d\phi\right)^2 + \\ &+\sqrt{h_1 h_5}\left(d\rho_1^2+\rho_1^2 d\psi^2+d\rho_2^2+\rho_2^2 d\phi^2\right) + \\ &+\sqrt{\frac{h_1}{h_5}}\sum_{i=6}^{9} dx^i dx^i\,. \end{aligned} \tag{6.10}$$

Comparing to [8], we see that $h_{1,5}$ are harmonic functions in the flat $\mathbf{R}^4$ of coordinates $(\rho_1, \psi, \rho_2, \phi)$, with sources on a circle at $\rho_1 = R$, $\rho_2 = 0$. These are then D1(z)-D5($z6789$) branes distributed on a tube.

In [8] it was found that an upper bound on the spin of supertubes follows from the requirement that ∂_ψ does not become timelike and hence CTCs do not appear. The place where this could happen is close to the tube, where the norm of ∂_ψ becomes approximately, up to a positive factor,[8]

$$g_{\psi\psi} \propto -J^2 + \frac{R_z^2}{g_s^4}Q_1 Q_5 R^2\,. \tag{6.11}$$

Then, the absence of CTCs implies an upper bound on the spin of the tube

$$J^2 \le \frac{R_z^2}{g_s^4}Q_1 Q_5 R^2 = \frac{R_z^2}{g_s^2}n_1 n_5 R^2\,, \tag{6.12}$$

or, using (6.6),

$$J \le \frac{n_1 n_5}{n_{\mathrm{KK}}}\,. \tag{6.13}$$

[8] See [8] for the expression including all terms (in this reference, $g_s = 1 = R_z$).

The extremal limit of the black ring solution satisfies (6.5) and (6.7), and hence does *not* saturate these bounds. As in [8], we interpret the solutions that do not saturate the spin bound as configurations where not all the D1 and D5 branes contribute to the rotation. Some of them are simply superimposed on the tube, together with those that contribute to the angular momentum. In the extremal limit, there are no forces between supertubes and (parallel) D1 or D5 branes, so the total energy of the configuration depends only on the total number of D1 and D5 branes, $M = \frac{\pi}{4G}(|Q_1| + |Q_5|)$.

If one fixes the conserved charges, or numbers of branes n_1 and n_5 in the solution, then the maximum possible spin is $J_{\rm max} = n_1 n_5$. Note that there are two different ways in which one can have $J < J_{\rm max}$. The extremal D1-D5 solutions where $J = n_1 n_5/n_{\rm KK}$ with $n_{\rm KK} \geq 1$ were studied in [10, 12], who showed that if $n_{\rm KK} = 1$ the geometry near the horizon is the non-singular global $\mathrm{AdS}_3 \times S^3$, while if $n_{\rm KK} > 1$ there is a conical deficit of the form $(\mathrm{AdS}_3 \times S^3)/\mathbf{Z}_{n_{\rm KK}}$ (see also [29]). Alternatively, one may have only a fraction of the branes contributing to the rotation, so the bound (6.13) is not saturated, as we have found for the extremal limit of the black ring. The cases where $J < n_1 n_5$ and $n_{\rm KK} = 1$ have been studied in [13, 14, 15]. In general, the solutions that do not saturate the spin bound (6.13), which include the extremal limit of the charged black ring, have a naked strong curvature singularity at the ring.

The D1-D5 supertubes with $J = n_1 n_5/n_{\rm KK}$ were constructed in [10] as a limit of the three-charge black hole. In this case the solutions near extremality had null singularities instead of horizons. In other words, this limit can not proceed through regular near-extremal black rings that are continuously connected to the supertube — the gap for excitations is too large. We are interested in approaching the supertubes through a sequence of non-extremal solutions with regular horizons, so that there exist near-extremal black rings arbitrarily close to the extremal, ground state supertube. The near-extremal solutions are then viewed as thermally excited states above the ground state. Let us first consider a sequence of equilibrium black rings, which have regular horizons. As the extremal limit is approached, we move toward supertubes with a specific value of J, which is half the maximum spin for fixed n_1, n_5 and $n_{\rm KK}$, (6.7). But if we allow for unbalanced black rings then we can obtain limiting supertubes with

$$J = \frac{\nu}{\lambda}\frac{n_1 n_5}{n_{\rm KK}}\,. \tag{6.14}$$

Regularity of the non-extremal black ring horizon, up to a possible conical singularity disk, requires $\nu < \lambda$. So we can find limiting supertubes with any spin below the CTC-bound (6.13) from (generically unbalanced) near-extremal black rings. The particular case where $\lambda \to \nu$ saturates the bound, although non-extremal black rings with $\lambda = \nu$ have strong naked singularities. Extremal tubes that exceed this bound are not connected to near-extremal black rings. It is not clear to us why only the supertube with precisely half the maximal spin can be excited to near-extremal black rings that are balanced. We are not even sure that the conical singularity disk of unbalanced rings can be given a meaning in the full string theory, but in the extremal limit these conical singularities always disappear due to a supersymmetric cancellation of forces.

7. Microscopic view: how strings tell a ball from a donut

For given D1 and D5 charges, there is a range of parameters (2.14) in which there exist both black holes and black rings that also have the same spin and mass. How can this duplicity of states be understood within a microscopic stringy interpretation?

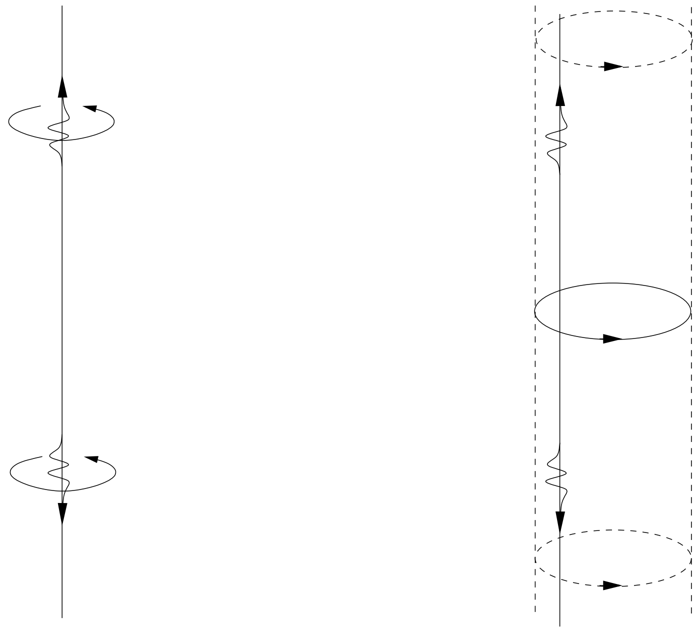
Let us first observe an important feature distinguishing D1-D5-charged black holes and black rings — recall that we are taking the momentum charge to be zero. The spin of the non-extremal D1-D5 black hole is bounded above by the energy above extremality. Near extremality the bound is

$$J^2 \leq \frac{\pi}{2G}(M - M_{\rm BPS})Q_1 Q_5\,, \tag{7.1}$$

with $M_{\rm BPS} = \frac{\pi}{4G}(|Q_1| + |Q_5|)$, and if this condition is violated then the near-extremal solutions become naked singularities. So the ground state of the D1-D5 spherical black holes (without momentum charge) is not rotating. As we will see, this reflects the fact that the angular momentum of the black hole is carried by the same excitations that lift the system above the BPS state. In contrast, we have seen that the spin of the black ring remains finite as the energy above extremality decreases to zero. The ground state of the spinning ring can have finite rotation. This indicates that the angular momentum in this case is carried in a way different than in spherical black holes.

In the following, we first identify the microscopic states that correspond to the extremal limits of the solutions, and then we consider their non-BPS excitations. As we have seen, the extremal BPS states are generically supersymmetric D1-D5 supertubes. Here we closely follow the quite intuitive description of them given in [11]. For simplicity we shall assume $n_{\rm KK} = 1$, but the number of KK-monopoles in the tube does not make much of a difference for our discussion of black hole non-uniqueness.

The dynamics of the bound state of n_1 D1-branes and n_5 D5-branes can be effectively described in terms of a number $n_1 n_5$ of strings along the common wrapped direction z of the D1 and D5 branes. These strings can be joined into a single string that is $n_1 n_5$ times longer. This single long "effective string" can be in several ground states, one of which has spin 1 (i.e., $(1/2, 1/2)$ of the rotation group in five dimensions $SU(2) \times SU(2) \simeq SO(4)$), a spin that is too small to register as a macroscopically large angular momentum. But the string can be excited by adding open strings that end on the D-branes and that propagate along the D1-D5 intersection. These modes can carry a circular polarization in the four non-compact directions and hence an angular momentum of SO(4). If there are a sufficiently large number of modes with polarizations that add coherently, then the string will possess a macroscopic rotation. When the open strings are all moving in the same direction, left or right, then the system is still supersymmetric and we obtain the microscopic description of the BMPV extremal rotating black hole with D1, D5, and momentum charges [17]. However, we are interested here in black holes without net linear momentum charge. In this case we can still give the system an angular momentum by exciting equal numbers of left and right movers, so the total linear momentum $P = (n_L - n_R)/R_z$ is zero, and orienting their polarizations in the same sense to give the system a macroscopic rotation. This configuration is depicted in figure 1a. Since we have excitations moving in opposite directions along the string, the state is no longer BPS. The supergravity description of this



(a) (b)

Figure 1: String theory interpretation of the spinning D1-D5 spherical black hole (a) and black ring (b). The vertical direction corresponds to the compactified sixth dimension, z, of the D1-D5 intersection. The wavy excitations up and down it are left- and right-moving open strings attached to the D1-D5 bound state. (a): In the spherical black hole, the angular momentum is carried by the polarization of (equal numbers of) left and right movers. When the left and right movers are turned off, the spin disappears. (b): The D1-D5 black ring is a tubular configuration where the angular momentum is due to the rotation around the tube of strings of the D1-D5 system. The effect of the left and right movers is only to excite the system above the BPS supertube ground state, so they can be switched off while keeping the angular momentum finite.

system is the non-extremal rotating black hole with D1 and D5 charges and a spherical horizon of finite area. It is the particular case of the solutions in [21, 22] where the momentum charge and the second angular momentum are zero, and is also obtained from our solutions with $\lambda = 1$ and zero momentum $\alpha_n = 0$. Close to extremality, its Bekenstein-Hawking entropy can be precisely reproduced by a state counting of the left and right moving modes [21]. It is obvious that, if we reduce to zero the number of left and right moving excitations, then the rotation disappears. This agrees with the property mentioned above that the spin of the D1-D5 spherical black hole decreases to zero as we approach the extremal limit.

There is, however, another way in which the D1-D5 state can acquire a macroscopic rotation. As we said, a single string can, by itself, carry only a unit of spin. But if a (macroscopic) fraction of the $n_1 n_5$ strings remain separate and do not join into a long string, then each of them can contribute a unit of spin, which add up to give a macroscopic angular momentum. These strings are still in a ground state, and they form a rotating supertube configuration. When *all* the $n_1 n_5$ strings are separate and all of them spin, then one obtains the supertube with maximal angular momentum $J_{\max} = n_1 n_5$. However, we have seen that this is not the state that corresponds to the extremal limit of the (equilibrium) black ring. Eq. (6.7) (with $n_{\rm KK} = 1$) shows that in the latter only $n_1 n_5/2$ of the effective strings contribute to the rotation, while the rest are superimposed on the tube but do not add to the spin. This is then the microscopic ground state of the D1-D5 black rings, where the branes themselves provide the rotation and the spin remains non-zero in the extremal limit.

We can also have supertubes with any spin $J \leq J_{\max}$ and therefore a different number of effective strings contributing the rotation, but, as we saw in (6.14), their associated near-extremal black rings are not balanced and present conical singularity disks. The microscopic explanation for this, and its consistency, are not yet clear to us. We shall center our discussion around equilibrium black rings, for which the issue of non-uniqueness is well-defined.

Having identified the microscopic string state for the extremal black ring, now one can excite it above the BPS ground state. We do not yet know the detailed nature of the thermal excitations of the ring, but it seems plausible that, if we do not want to give the ring any net linear momentum charge, then we can simply add equal numbers of left and right moving open string excitations, see figure 1*b*. The open string modes are presumably carried by the effective strings that do not contribute to the angular momentum, and which should join into a string $n_1 n_5/2$ times longer so as to lower the energy gap of excitations and maximize the entropy. These excitations can be switched off smoothly while the system retains its angular momentum, and this is indeed what we observe in the supergravity solutions. We hope to clarify the nature of these non-BPS excitations in subsequent work, but at present we only need the fact that near-extremal black rings correspond, at weak coupling, to thermally excited supertubes.

So string theory assigns different microscopic configurations to spherical black holes and black rings. The identification of states that we have performed is under good control if the systems remain close to their supersymmetric ground state. This implies that the supergravity solutions must be close to extremality, i.e., ν close to zero, far from the range of ν (2.15) where the non-uniqueness of the solutions is apparent. But it is natural to assume that, as we move further away from the BPS state by adding more thermal excitations, the corresponding supergravity solution changes continuously and preserves its topology. Then, spherical black holes and black rings are still in correspondence with thermally excited states of a single long D1-D5 string, and of supertubes, respectively. In this way, string theory can resolve a black hole and a black ring with the same charges, spin and mass.

It remains unclear yet how string theory distinguishes between the two black rings that also exist with the same asymptotic charges.[9] As we move away from extremality, say, increasing the mass for fixed spin and charges, the second black ring appears initially as a singular solution, and always has less entropy than the first black ring (see figure 3 in [2]). So the second solution seems to be a metastable excitation of the system. In order to understand its complete significance we may need a more detailed knowledge of the system far from extremality than we currently have.

[9]We may say that we are not able yet to distinguish a donut from a bagel.

8. Discussion

The addition of charge to the solutions of [2] has provided us with a connection between two recently discovered types of rotating objects in (super)gravity: black rings [2] and supertubes [8, 10, 12]. Thus we have been able to recognize non-extremal black rings as thermal excitations of supertubes, and this permits the identification of black rings with states of string theory. An essential point in this identification is that the different topologies of black rings and black holes correspond to different topologies of the D-brane configurations obtained at weak coupling. So string theory contains the required states to account for at least some of the new supergravity solutions, and this implies that the notion of black hole uniqueness is not a necessary ingredient for the success in the microscopic account of black hole entropy.

We have presented our discussion of the stringy description of black holes and black rings in section 7 in the rather pictorial terms of the "effective string model", which is both convenient and sufficient for identifying the origin of non-uniqueness. It is also possible to employ the more precise language of the two-dimensional conformal field theory that is dual to the D1-D5 system. The states of the conformal theory that correspond to the extremal D1-D5 solutions are, under spectral flow from the R to the NS sector, a particular class of chiral primaries of the theory, and have been analyzed in detail in [13, 14, 15, 29]. The near-extremal black rings also possess a near-horizon geometry that is asymptotically $\mathrm{AdS}_3 \times S^3$, and we are currently investigating their dual CFT description. We hope to report on this elsewhere.

A surprising feature of three-charge D1-D5-P black rings is their deadly pathological nature. We have seen that this is due to the incompatibility of the KK-monopole structure, present whenever there are both D1 and D5 branes rotating in a tube, and the boost that gives momentum to the solution. An explanation of this issue from the AdS/CFT perspective should clarify at the microscopic level why, if the thermal excitations in the near-extremal black ring are momentum-carrying open strings, one can have $n_L = n_R$ while $n_L \neq n_R$ seems to be more difficult or even inconsistent.

We have illustrated this pathology with the example (5.12) of the combination of a KK-monopole and a boost in the KK-direction, but this example does not imply that there is no consistent solution with both electric and magnetic KK charges. Such solutions are actually known [30], and they are free from the problems of (5.12). There is also some understanding of them within string theory [31]. However, it is unclear whether it is possible to add momentum to the D1-D5 bound state in a KK-monopole tube in a similar manner. It may also be interesting to see if a worldvolume analysis of the dual system with D0-F1-D4 charges is able to shed further light on this issue.

The concept of black hole uniqueness in four dimensions is well-defined only for non-extremal solutions, with regular non-degenerate horizons. The extremal limits of our charged black rings are 1/4-supersymmetric solutions whose horizons, even in the cases when they are non-singular, are degenerate, and moreover have zero area. So even if a large number of these solutions do all have the same asymptotic charges,[10] they do not

[10] Classically, an infinite number labeled by continuous parameters. Quantization should discretize the

pose any problem to the classical notions of uniqueness or no-hair. Recently, a uniqueness theorem has been proven for supersymmetric black holes (with a degenerate horizon) of minimal supergravity in five dimensions [3], but our two-charge solutions do not belong in this theory, and furthermore they are typically singular. Nevertheless, they are physically acceptable since they can be put in correspondence with string states [14, 15].

Questions like the interpretation of the non-uniqueness presented by the duplicity of black rings remain open and beyond the control of the near-extremal regime. Another similarly unresolved issue concerns the lower bound on the spin of the ring. It seems that, if we add energy to the ring while keeping its spin and charges fixed, there is a point at which the ring cannot support itself against collapse any longer and becomes unstable. Beyond this point there are no black ring solutions. As shown in [16], charge provides a repulsive force that allows for equilibrium at smaller spins for a given mass, but there still remains a lower bound on J^2/M^3. In fact, stability is an important open issue for black rings (indeed, for all rotating black holes in more than four dimensions [4]). Since thin black rings look locally like (boosted) black strings, they are expected to suffer from the Gregory-Laflamme instability [32]. The addition of charge, however, may increase their stability, and perhaps near-extremal black rings slightly excited above the BPS supertube ground state are stable.

While we can hardly expect that present techniques provide a detailed understanding of non-uniqueness far from extremality, and in particular for neutral black holes, we may still get some clues from what we have learned so far. First of all, string theory does seem to possess the additional states required to account for ring-like configurations. Second, topology has been of crucial help in the matching to string states: as the coupling is decreased, charged black holes and black rings appear to go over into string configurations which themselves have different topology, figure 1. The open string excitations make the D1-D5 brane bound state somewhat fuzzier, and give a thickness to the D1-D5 strings sketched in figure 1. As one moves further away from extremality this fuzziness increases and becomes a dominant feature. So this suggests a picture, presumably the simplest one, where the topology of the solutions is preserved across the correspondence point. Neutral rotating black holes should correspond to highly excited (random-walk) strings that oscillate rapidly about a center, and thus take the shape of a fuzzy string ball. If the spin of these oscillations does not average to zero we get a net angular momentum. Black rings, instead, would correspond to a less conventional state where the excited string is wiggling about a circle and so has a fuzzy donut-like shape. The angular momentum is carried by the coherent orbital rotation of the string-donut around the circle, and it prevents the state from collapsing into a string ball. We are not aware of any previous discussion of such a string-donut state, but it should be interesting to investigate whether it can exist, perhaps as a less stable configuration than the string ball. String balls appear at weak coupling as poles in string scattering amplitudes (see e.g., [33]), but it is not clear how to find string donuts as intermediate states.

Of course this is a very rough picture. It is far from obvious whether it can explain issues such as the dimensionality dependence of these effects, and the upper and lower

spectrum.

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bounds on the spin of black holes and rings — recall that the spin of fundamental string states is bounded above $J \leq \alpha' M^2$ in any dimension. Moreover, topology cannot resolve the duplicity among black rings. It will be interesting to investigate whether we can at least obtain qualitative answers to these questions.

We have considered configurations where the "effective string", i.e., the D1-D5 intersection, is transverse to the ring circle, thus forming a tube. However, one may envisage a different situation where the effective string is instead bent into a circle to form itself the ring. In that case, there would not be any net D1 and D5 charges, only dipole sources, and the momentum that runs along the effective string would correspond to the angular momentum. Such configurations, even when they are extremal, are not expected to be supersymmetric. Rather than the D1-D5 bound state, which would have to be smeared in one of the compact directions transverse to a five-dimensional ring, it would be more interesting to consider other systems that also yield effective strings and which are more localized, such as the intersection of three M5-branes over a string [34], or any of its U-duals. One such configuration was discovered even before the neutral black ring was found. Ref. [35] described a solution of 11D supergravity where three M5-branes intersect over a string loop, i.e., a ring. The ring is not balanced, since it is a static solution and there is no momentum running along the string loop. Hence it has a conical singularity, but besides this, its near-horizon geometry was shown to be AdS_3 times a distorted S^2.[11] The addition of a certain amount of angular momentum should balance the ring and presumably eliminate the distortion, and yield an extremal ring with a regular horizon of finite area. Since this solution would not have any asymptotic conserved charges other than mass and spin, it would provide an infinite-fold violation of black hole uniqueness,[12] a possibility also pointed out in [3]. So it would be quite interesting to find it.

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A. Towards the three-charge solution

In this appendix we first give the solution for the black tube with charges [$P(z)$,F1(z)]. Applying a sequence dualities, we then obtain a black tube with charges [$P(z)$, F1(z), NS5($z6789$)], and reduction of this to five dimensions yields the three-charge spinning black ring of section 3.

[11] When the ring is balanced by immersing it in a fluxbrane, the conical singularity and the distortion disappear and the geometry near the horizon is exactly $\mathrm{AdS}_3 \times S^2$ [35].

[12] See footnote 10.

The starting point is the ten-dimensional vacuum solution obtained by adding to the black ring solution (2.1) a flat direction z as well as four directions x^i, $i = 6, 7, 8, 9$, on a torus T^4. We apply a boost with parameter α_5 in the z-direction, T-dualize z, and then apply a second boost with parameter α_1. The resulting solution with $P(z)$ and F1(z) charges associated with α_1 and α_5, respectively, is described by the string-frame metric

$$\begin{aligned} ds_6^2 = & -\frac{F(x)\hat{h}_1(x,y)}{F(y)h_5(x,y)}dt^2 + \frac{(x-y)\lambda\sinh 2\alpha_1}{F(y)h_5(x,y)}dtdz + \frac{h_1(x,y)}{h_5(x,y)}dz^2 + \\ & + \frac{2\sqrt{\lambda\nu}R(1+y)F(x)\cosh\alpha_5\cosh\alpha_1}{F(y)h_5(x,y)}dtd\psi + \\ & + \frac{2\sqrt{\lambda\nu}R(1+y)F(x)\cosh\alpha_5\sinh\alpha_1}{F(y)h_5(x,y)}dzd\psi - \frac{F(x)\lambda\nu(y+1)^2R^2\cosh^2\alpha_5}{F(y)h_5(x,y)}d\psi^2 + \\ & + \frac{R^2}{(x-y)^2}\left[-F(x)\left(G(y)d\psi^2 + \frac{F(y)}{G(y)}dy^2\right) + F(y)^2\left(\frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right)\right]. \end{aligned} \quad \text{(A.1)}$$

where $h_i(x,y)$ was defined in (3.6), and $\hat{h}_i(x,y) \equiv h_i(y,x)$. The full ten-dimensional solution also includes four flat directions, $g_{ij} = \delta_{ij}$ for $i = 6, 7, 8, 9$.

The dilaton is given by

$$e^{-2\Phi} = h_5(x,y)\,, \quad \text{(A.2)}$$

and the antisymmetric tensor field has the following nonzero components:

$$B_{t\psi} = -\frac{\sqrt{\lambda\nu}RF(x)(1+y)\sinh\alpha_5\sinh\alpha_1}{F(y)h_5(x,y)}\,, \quad \text{(A.3)}$$

$$B_{tz} = \frac{(x-y)\lambda\sinh 2\alpha_5}{2F(y)h_5(x,y)} \quad \text{(A.4)}$$

$$B_{\psi z} = \frac{\sqrt{\lambda\nu}RF(x)(1+y)\cosh\alpha_1\sinh\alpha_5}{F(y)h_5(x,y)}\,. \quad \text{(A.5)}$$

Applying to the $[P(z)$, F1$(z)]$ solution the sequence of dualities

$$\mathrm{S} + \mathrm{T}(6789) + \mathrm{S} + \mathrm{T(z)} + \text{boost } \alpha_n \quad \text{(A.6)}$$

we obtain the $[P(z)$,F1(z),NS5$(z6789)]$ solution. For each step, we outline the result of the dualities and list the types of charges:

S-dualize I: $\rightarrow [P(z)$, D1$(z)]$

Metric: $ds'^2 = h_5^{1/2}(ds_6^2 + dx^i dx^i)$, with ds_6^2 given in (A.1).

Dilaton: $e^{-2\Phi'} = h_5^{-1}$.

Forms: $C'^{(2)}_{\mu\nu} = -B_{\mu\nu}$, with $B_{\mu\nu}$ given in (A.3), (A.4), and (A.5).

T(6789): $\to [P(z),\, \mathrm{D5}(z6789)]$

Metric: $ds''^2 = h_5^{1/2} ds_6^2 + h_5^{-1/2} dx^i dx^i$.

Dilaton: $e^{-2\Phi''} = h_5$.

Forms: $C''^{(6)}_{\mu\nu 6789} = C'^{(2)}_{\mu\nu} = -B_{\mu\nu}$. For the next step we need instead of $C''^{(6)}$ the Hodge dual 2-form potential, $dC''^{(2)} = \star dC''^{(6)}$. We find

$$C''^{(2)}_{t\phi} = (1+x)R\sqrt{\lambda\nu}\sinh\alpha_5\cosh\alpha_1\,, \tag{A.7}$$

$$C''^{(2)}_{\phi z} = -(1+x)R\sqrt{\lambda\nu}\sinh\alpha_5\sinh\alpha_1\,, \tag{A.8}$$

$$C''^{(2)}_{\psi\phi} = -\frac{1}{2}R^2\lambda\sinh 2\alpha_5\left(\frac{G(x)}{x-y} + k(x)\right), \tag{A.9}$$

where $k(x) = x(1+\nu) - \nu x^2 + \text{const}$. We have fixed the constants of integration so that $C''^{(2)}_{t\phi}$ and $C''^{(2)}_{\phi z}$ go to zero at infinity.

S-dualize II: $\to [P(z),\, \mathrm{NS5}(z6789)]$

Metric: $ds'^2 = h_5 ds_6^2 + dx^i dx^i$.

Dilaton: $e^{-2\Phi'''} = h_5^{-1}$.

Forms: $B'''_{\mu\nu} = C''^{(2)}_{\mu\nu}$, with $C''^{(2)}_{\mu\nu}$ given in (A.7), (A.8), and (A.9).

T(z) and boost α_n: $\to [P(z),\, \mathrm{F1}(z),\, \mathrm{NS5}(z6789)]$

Metric: In the full ten-dimensional solution, there are four flat directions compactified on T^4: $g_{ij} = \delta_{ij}$ for $i,j = 6,7,8,9$. We can write the six-dimensional metric in a form appropriate for KK reduction along the z coordinate (string frame):

$$\begin{aligned} ds^2 = {} & \frac{h_n(x,y)}{h_1(x,y)}\bigg(dz + \frac{(x-y)\lambda\sinh 2\alpha_n}{2F(y)h_n(x,y)}\,dt + \\ & \qquad + \sqrt{\lambda\nu}R(1+y)\frac{F(x)}{F(y)h_n(x,y)}\cosh\alpha_5\cosh\alpha_1\sinh\alpha_n\,d\psi - \\ & \qquad - \sqrt{\lambda\nu}R(1+x)\frac{1}{h_n(x,y)}\sinh\alpha_5\sinh\alpha_1\cosh\alpha_n\,d\phi\bigg)^2 - \\ & - \frac{1}{h_1(x,y)h_n(x,y)}\frac{F(x)}{F(y)}\bigg(dt - \sqrt{\lambda\nu}R(1+y)\cosh\alpha_5\cosh\alpha_1\cosh\alpha_n\,d\psi + \\ & \qquad + \sqrt{\lambda\nu}R(1+x)\sinh\alpha_5\sinh\alpha_1\sinh\alpha_n\,d\phi\bigg)^2 + \\ & + h_5(x,y)\frac{R^2}{(x-y)^2}\bigg[-F(x)\left(G(y)d\psi^2 + \frac{F(y)}{G(y)}dy^2\right) + \\ & \qquad + F(y)^2\left(\frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right)\bigg]. \end{aligned} \tag{A.10}$$

Dilaton:

$$e^{-2\tilde{\Phi}} = \frac{h_1(x,y)}{h_5(x,y)} \,. \tag{A.11}$$

Forms:

$$\tilde{B}_{t\psi} = -\frac{\sqrt{\lambda\nu}R(1+y)F(x)\cosh\alpha_5\sinh\alpha_1\sinh\alpha_n}{F(y)h_1(x,y)} \tag{A.12}$$

$$\tilde{B}_{t\phi} = \frac{\sqrt{\lambda\nu}R(1+x)\sinh\alpha_5\cosh\alpha_1\cosh\alpha_n}{h_1(x,y)} \tag{A.13}$$

$$\tilde{B}_{\psi\phi} = -\frac{1}{2}R^2\lambda\sinh 2\alpha_5\left(\frac{G(x)}{x-y}+k(x)+\frac{\nu F(x)(1+x)(1+y)\sinh^2\alpha_1}{F(y)h_1(x,y)}\right) \tag{A.14}$$

$$\tilde{B}_{tz} = \frac{(x-y)\lambda\sinh 2\alpha_1}{2F(y)h_1(x,y)} \tag{A.15}$$

$$\tilde{B}_{\psi z} = \frac{\sqrt{\lambda\nu}R(1+y)F(x)\cosh\alpha_5\sinh\alpha_1\cosh\alpha_n}{F(y)h_1(x,y)} \tag{A.16}$$

$$\tilde{B}_{\phi z} = -\frac{\sqrt{\lambda\nu}R(1+x)\sinh\alpha_5\cosh\alpha_1\sinh\alpha_n}{h_1(x,y)} \,. \tag{A.17}$$

The solution with D1, D5, and P charges, in string frame, is obtained by multiplying (A.10) by $\sqrt{h_1(x,y)/h_5(x,y)}$, inverting the dilaton $\tilde{\Phi} \to -\tilde{\Phi}$ and S-dualizing the NS-NS two-form to an RR two-form. The three boosts α_5, α_1, α_n are each associated to the D5, D1, and P charges, respectively.

B. BMPV black hole as an extremal limit

In section 6 we mentioned a way to take the extremal limit which involves sending $R \to 0$, while keeping $M_0 e^{2\alpha_i}$, i.e., Re^{α_i}, finite. Since R sets the scale for the metric, we have to take the limit in such a way that we blow up (a part of) the geometry to finite size. The change of coordinates

$$x = -1 + \frac{2(1+\lambda)^2}{1+\nu}\frac{R^2\cos^2\theta}{r^2}\,, \qquad y = -1 - \frac{2(1+\lambda)^2}{1+\nu}\frac{R^2\sin^2\theta}{r^2}\,, \tag{B.1}$$

does precisely this: when $R \to 0$ keeping r and θ finite, the region near $x, y \simeq -1$ is blown up. The topology of the final solution need not be the same as the original one, and in fact the global problems that arise for black rings due to the fixed-point set of ∂_ϕ at $x = +1$ disappear since $x = +1$ is pushed away to infinity. Hence this limit can be considered even for three-charge black rings, which as we saw in section 5 are otherwise pathological.

The ten-dimensional extremal D1-D5-P solution that one obtains is

$$\begin{aligned} ds^2 = & \frac{h_n}{\sqrt{h_1 h_5}}\left(dz + \frac{Q_n}{h_n r^2}dt - \frac{g_s^2}{R_z}\frac{J\sin^2\theta}{h_n r^2}\,d\psi - \frac{g_s^2}{R_z}\frac{J\cos^2\theta}{h_n r^2}\,d\phi\right)^2 - \\ & -\frac{1}{h_n\sqrt{h_1 h_5}}\left(dt + \frac{g_s^2}{R_z}\frac{J\sin^2\theta}{r^2}\,d\psi + \frac{g_s^2}{R_z}\frac{J\cos^2\theta}{r^2}\,d\phi\right)^2 + \\ & +\sqrt{h_1 h_5}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\psi^2 + \cos^2\theta d\phi^2\right)\right] + \sqrt{\frac{h_1}{h_5}}\sum_{i=6}^{9} dx^i dx^i\,, \end{aligned} \tag{B.2}$$

with ψ and ϕ rescaled to have periodicities 2π, and

$$h_i = 1 + \frac{Q_i}{r^2}\,. \tag{B.3}$$

This is the extremal D1-D5-P spherical rotating black hole, which upon reduction yields the five-dimensional BMPV black hole [17, 18]. Note that we obtain this solution regardless of the values of λ and ν, i.e., of whether we are taking the limit of a sequence of black holes or black rings. However, the limits of black holes and black rings result into different ranges of values of the final parameters. If all three charges are preserved in the limit, then, we obtain the spin as

$$J^2 = \left(\frac{27\pi}{32G}\frac{J_0^2}{M_0^3}\right) n_1 n_5 n_L\,. \tag{B.4}$$

Along a sequence of black holes we get spins in the range $0 \leq J^2 \leq n_1 n_5 n_L$, for which there are no CTCs outside the black hole horizon. If instead we take an equilibrium black ring sequence, with $\lambda = \lambda_c$, then for $0 < \nu \leq \sqrt{5} - 2$ we get spins $J^2 \geq n_1 n_5 n_L$, which is the over-rotating regime characterized by the appearance of CTCs outside the horizon. As extremality is approached, these CTCs do not have their origin in the periodic identification of time (5.3), but rather on ψ becoming timelike outside the horizon. It is also clear from (B.4) that there is a range of parameters for which a black hole and two black rings limit to the same (under-rotating) black hole at extremality.

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